

1. In order to express $\sum_{n=-\infty}^{\infty} d_n e^{inx} = \sum_{n=-\infty}^{\infty} c_{2n} e^{inx}$ in terms of f , first we will "get rid of" the odd indexed c_n 's in $\sum_{n=-\infty}^{\infty} c_n e^{inx}$, i.e. we will express $\sum_{n=-\infty}^{\infty} c_{2n} e^{ix2n}$ with the help of $f(x)$.

$$\begin{aligned} \sum_{n=-\infty}^{\infty} c_{2n} e^{ix2n} &= \frac{\sum_{n=-\infty}^{\infty} c_n e^{inx} + \sum_{n=-\infty}^{\infty} (-1)^n c_n e^{inx}}{2} = \\ &= \frac{f(x) + \sum_{n=-\infty}^{\infty} c_n e^{in(x+\pi)}}{2} = \\ &= \frac{f(x) + f(x+\pi)}{2}. \end{aligned}$$

A change of variable, $y := 2x$, gives

$$\sum_{n=-\infty}^{\infty} c_{2n} e^{iny} = \frac{f(y/2) + f(y/2 + \pi)}{2}.$$

□