Universal Enveloping Algebra of a Lie Algebra

$$U(L) = T(L)/_{ab-ba=[a,b]}$$

Associative Algebra over the field $\mathbb{C}$

- Multiplication $m : \mathcal{A} \otimes \mathcal{A} \to \mathcal{A}$

$$m(1 \otimes m) = m(m \otimes 1)$$

- Unit $\eta : \mathbb{C} \to \mathcal{A}$

Co-Algebra

- Co-multiplication $\Delta : \mathcal{A} \to \mathcal{A} \otimes \mathcal{A}$

$$(\Delta \otimes 1)\Delta = (1 \otimes \Delta)\Delta$$

- Co-unit $\epsilon : \mathcal{A} \to \mathbb{C}$

Bialgebra $(\mathcal{A}, m, \eta, \Delta, \epsilon)$
Compatibility conditions: $\Delta(ab) = \Delta(a)\Delta(b)$, $\epsilon(ab) = \epsilon(a)\epsilon(b)$

Hopf Algebra: Bialgebra + antipode
Antipode $\gamma : \mathcal{A} \to \mathcal{A}$

$$\gamma(ab) = \gamma(b)\gamma(a)$$

$$m(\gamma \otimes 1)\Delta = m(1 \otimes \gamma)\Delta = \eta\epsilon$$

$U[\mathfrak{sl}(2)]$ $\{X, Y, H\}$

$$[X, Y] = H$$

$$[H, X] = 2X$$

$$[H, Y] = -2Y$$

$U_q[\mathfrak{sl}(2)]$ $\{E, F, K, K^{-1}\}$, $q \in \mathbb{C}$

$$KK^{-1} = K^{-1}K = 1$$

$$KEK^{-1} = q^2 E$$

$$KFK^{-1} = q^{-2} F$$

$$[E, F] = \frac{K - K^{-1}}{q - q^{-1}}$$

$U-$module of $U_q[\mathfrak{sl}(2)]$
Highest weight vector u

$$Ku = \lambda u$$

$$Eu = 0$$

Rest eigenvectors u_p

$$u_p = \frac{1}{[p]!} F^p u,\ p > 0 \text{ and } [p]_q := \frac{q^p - q^{-p}}{q - q^{-1}}$$

$$Ku_p = \lambda q^{-2p} u_p$$

$$Eu_p = \frac{q^{-(p-1)} - q^{p-1}\lambda^{-1}}{q - q^{-1}} u_{p-1}$$

$$Fu_p = [p] u_{p+1}$$