More About the Yang-Baxter Equation

- By doing the following change in notation:

$$R''_{ab} \to R_{12} \qquad R_{ai} \to R'_{13} \qquad R'_{bi} \to R''_{23}$$

The Yang-Baxter equation looks like:

$$\boxed{R_{12}R'_{13}R''_{23} = R''_{23}R'_{13}R_{12}}$$

In components:

$$\sum_{j_1,j_2,j_3} R^{k_1k_2}_{j_1j_2} R'^{j_1k_3}_{i_1j_3} R''^{j_2j_3}_{i_2i_3} = \sum_{j_1,j_2,j_3} R''^{k_2k_3}_{j_2j_3} R'^{k_1j_3}_{j_1i_3} R^{j_1j_2}_{i_1i_2}$$

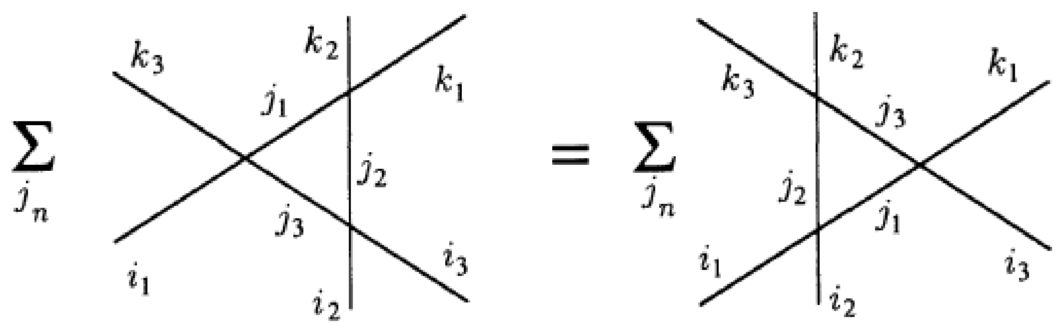

*Image taken from [GRS]

- Using the parametrization:

$a = \rho \sinh(\lambda + \phi)$ $\quad$ $b = \rho \sinh(\lambda)$ $\quad$ c= $\rho \sinh(\phi)$

$$\Delta = \cosh \phi$$

We can parametrize $R = R(\lambda)$, $R' = R(\lambda')$ and $R'' = R(\lambda'')$

- The Yang-Baxter equation is now:

$$\boxed{R_{12}(\lambda)R_{13}(\lambda + \lambda')R_{23}(\lambda') = R_{23}(\lambda')R_{13}(\lambda + \lambda')R_{12}(\lambda)}$$

- RTT=TTR relation in components:

$$\sum_{j_1,j_2} R(\lambda-\lambda')^{k_1 k_2}_{j_1 j_2} T(\lambda)^{j_1}_{i_1} T(\lambda')^{j_2}_{i_2} = \sum_{j_1,j_2} T(\lambda')^{k_2}_{j_2} T(\lambda)^{k_1}_{j_1} R(\lambda-\lambda')^{j_1 j_2}_{i_1 i_2}$$

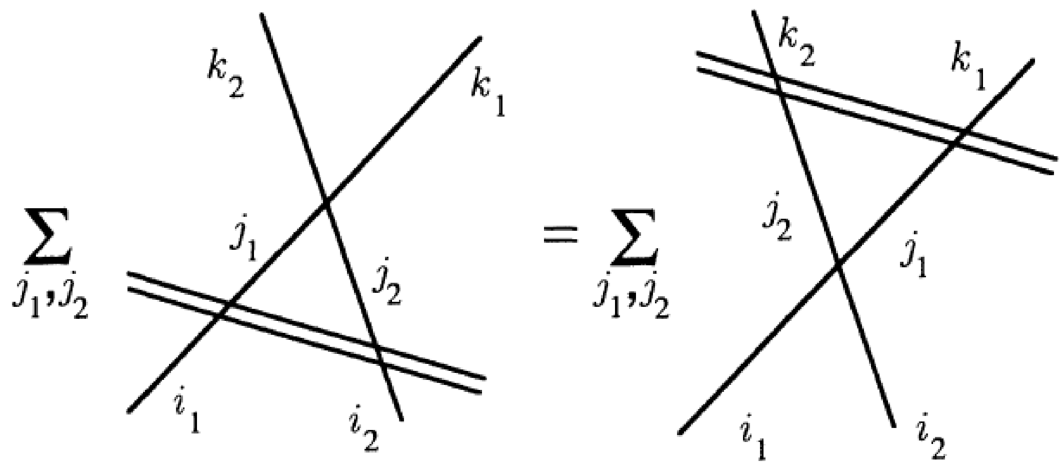

*Image taken from [GRS]

- Similar to the Yang Baxter equation for the R matrices.

Liouville's Theorem

- If a system with a phase-space of $2n$ dimensions, has n functions F_i such that:

$$\{F_i, F_j\}_{P.B.} = 0$$

The system can be solved by quadratures.

References

- **[Aru]** Arutyunov: Student Seminar, Classical and Quantum Integrable Systems. Lecture notes (2007)
- **[Fra]** Franchini: Notes on Bethe Ansatz Techniques. Lecture notes (2011)
- **[GRS]** Gómez, Ruiz-Altaba, Sierra: Quantum Groups in Two-Dimensional Physics. Cambridge University Press (1996)
- **[JM]** Jimbo, Miwa: Algebraic Analysis of Solvable Lattice Models. American Mathematical Society (1995)