Baxter's TQ-construction
An alternative to the Bethe Ansatz

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Schedule

- Baxter's TQ-Construction
- Application of TQ to the 6-vertex model
- (If there is time) Link to Bethe Ansatz results
- Extension to 8-vertex model

Baxter's TQ-Construction for the 6-vertex model

- Given a Transfer Matrix $\mathbf{T}$ belonging to Boltzmann weights a, b and c we can find other transfer matrices **T'** with corresponding Boltzmann weights a', b' and c' so that $[\mathbf{T}, \mathbf{T'}] = 0$. This will impose a (familiar) condition on the Boltzmann weights

$$\frac{a^2 + b^2 - c^2}{2ab} = \Delta = \Delta' = \frac{a'^2 + b'^2 - c'^2}{2a'b'}$$

- A forteriori, we can reparameterize a, b and c in terms of a scaling factor ρ and rapidities λ and ϕ. For a fixed choice of Δ these transfer matrices form a 1-parameter family $\mathcal{T} = \{\mathbf{T}(\lambda) | \lambda \in \mathbb{C}\}$. All elements of a given $\mathbf{T}(\lambda)$ are entire functions of λ (the spectral parameter).

- There exists a matrix function $\mathbf{Q}(\lambda)$ that satisfies the commutation relations

$$[\mathbf{Q}(\lambda), \mathbf{Q}(\lambda')] = [\mathbf{Q}(\lambda), \mathbf{T}(\lambda')] = 0$$

  and the TQ-relation

$$\mathbf{T}(\lambda)\mathbf{Q}(\lambda) = \sigma(\lambda - \phi)\mathbf{Q}(\lambda + 2\phi) + \sigma(\lambda + \phi)\mathbf{Q}(\lambda - 2\phi)$$

  Where $\sigma(\lambda) = [\rho \sinh \lambda]^L$

- The determinant of $\mathbf{Q}(\lambda)$ is not entirely zero, and it decomposes into diagonal blocks on each subspace of fixed magnon number M.

- We recover the Bethe Ansatz and can now calculate the eigenvalues $\Lambda(\lambda)$ of $\mathbf{T}(\lambda)$.

Extension to 8-vertex model

- Given a Transfer Matrix **T** belonging to Boltzmann weights a, b, c and d we can find other transfer matrices **T'** with corresponding Boltzmann weights a', b', c' and d'so that $[\mathbf{T}, \mathbf{T'}] = 0$. This will impose a conditions on the Boltzmann weights

$$\frac{a^2 + b^2 - c^2 - d^2}{2(ab + cd)} = \Delta = \Delta' = \frac{a'^2 + b'^2 - c'^2 - d'^2}{2(a'b' + c'd')}$$
$$\frac{ab - cd}{ab + cd} = \Gamma = \Gamma' = \frac{a'b' - c'd'}{a'b' + c'd'}$$

- A forteriori, we can reparameterize a, b and c in terms of a scaling factor ρ and three other numbers λ, κ and ϕ. For a fixed choice of Δ and Γ these transfer matrices form a 1-parameter family $\mathcal{T} = \{\mathbf{T}(\lambda) | \lambda \in \mathbb{C}\}$. All elements of a given $\mathbf{T}(\lambda)$ are entire functions of λ (the spectral parameter).

- There exists a matrix function $\mathbf{Q}(\lambda)$ that satisfies the commutation relations

  $$[\mathbf{Q}(\lambda), \mathbf{Q}(\lambda')] = [\mathbf{Q}(\lambda), \mathbf{T}(\lambda')] = 0$$

  and the TQ-relation

  $$\mathbf{T}(\lambda)\mathbf{Q}(\lambda) = \sigma(\lambda - \phi)\mathbf{Q}(\lambda + 2\phi) + \sigma(\lambda + \phi)\mathbf{Q}(\lambda - 2\phi)$$

  Where $\sigma(\lambda)$ is a significantly more complicated function.

- The determinant of $\mathbf{Q}(\lambda)$ is not entirely zero, and it decomposes into diagonal blocks on the subspace of even and the subspace of odd magnon numbers.

- We recover equations similar to the Bethe Ansatz and can now calculate the eigenvalues $\Lambda(\lambda)$ of $\mathbf{T}(\lambda)$.